

CORRELATIONAL ANALYSIS BETWEEN APPROXIMATE NUMERICAL ESTIMATE AND MATH ABILITIES: A STUDY WITH FIRST-GRADE SCHOOL STUDENTS

Eduardo Guimarães Mielo¹, & Paulo Sérgio Teixeira do Prado²

¹Undergraduate in Pedagogy, São Paulo State University - Unesp (Brazil)

²Department of Education and Human Development, São Paulo State University – Unesp (Brazil)

Abstract

The number sense has two characteristics: subitizing – the immediate and error-free recognition of numbers from one to three, without counting – and the ability to discriminate between numbers with values greater than the subitizable ones. Without counting, this type of discrimination is done by approximate estimate, from which a numerical approximation system is inferred. Although the approximate numerical estimate is considered innate, it is influenced by factors such as the ratio between the sets to be compared, external factors, in addition to its increasing accuracy with age. This cognitive ability has been identified as a “predictor” for academic achievement in mathematics. The presented research aimed to investigate the existence of a correlation between the approximate number estimate and math skills in first-grade school children. The study was carried out with 34 children aged between six and seven years old. Their math skills were measured using the Test of Early Mathematical Abilities (TEMA-3), which was administered following the protocol, designed to measure general formal and non-formal math skills of children from 3 to 8 years-old. The approximate number estimate was measured using *Panamath* – a software that managed the discrimination task –, consisting of comparisons of several pairs of sets, indicating the largest. The exposure time of the stimuli was set at 1200 ms and the ratio of the difference between the sets was systematically varied. The results showed an ease of distinction based on the proportions of the sets in the numerical approximation system test. This study investigated the possibility of a correlation between performance in the number sense activity with those of mathematical skills, as it is assumed that the greater the child's sensitivity to differentiating the proportions between sets, the better his performance in solving the mathematical problems addressed, accordingly to previous studies. Even though the Pearson's correlation coefficient was 0.31 ($p = 0.07$, a little higher than accepted), the value indicates a moderate to weak correlation and a possible prediction in mathematical abilities based on performance in the numerical discrimination task, although there are other variables to consider in the mathematical development. The approximate number system test can be used as a tool to do and initial track of children who might experience problems in developing math skills.

Keywords: Numerical estimation, number sense, math abilities, counting.

1. Introduction

Number sense has two defining characteristics. One of them is the prompt and error-free recognition of numbers from one to three, without the use of counting or other linguistic resources. Given the fact that this recognition occurs in fractions of a second – that is, suddenly – it is designated by the term “subitization” (Mandler; Shebo, 1982). The other signature of the number sense is the ability to discriminate between numbers with values greater than the subitizable ones (four or more).

For Corso and Dorneles (2010), it is estimated that the number sense is understood through the child's ability to relate to numbers and understand ideas related to them. The number sense, as well as the mathematical skills that the child develops throughout his life, is composed of a set of factors, such as awareness, intuition, recognition, process, conceptual structure, mental number line, counting, comparison and ordering, composition/decomposition and addition/subtraction (Berch, 2005; Sarama & Clemens, 2009). For Nunes and Bryant (1997), this concept about number sense can be related to numeralization, which brings the individual closer to numbers and their abilities to develop daily activities, in addition to using mathematics as a form of communication.

Although Feigenson, Dehaene and Spelke (2004) define a concept of number sense, it is stated that the representation of non-symbolic numbers can help in the resolution of more complex mathematical

expressions as the school years progress. When this type of discrimination is done without counting, it is a so-called rough estimation skill, which allows the individual to differentiate between two sets, identifying the "larger", that is, the one with more items. However, there are other factors involved in the comparison between sets, such as the total area occupied by their elements, the way they are distributed in space and others (Leibovich et al., 2017; Mix; Huttenlocher; Levine, 2002).

The Approximate Numerical System (ANS), unlike counting, is a form of non-symbolic numerical representation and, therefore, more imprecise. However, it is a feature that allows for set discrimination. Some factors that influence it were identified, for example, the ratio between the sets to be compared (Moyer & Landauer, 1967). The greater the difference between them, the easier the discrimination. The ANS has been identified by some researchers as a "predictor" for school performance in mathematics (Mazzocco; Feigenson; Halberda, 2011). However, it is not the only predictor. Studies show other factors, such as the mother's education (Koponen, 2007), preschool education (Arnold, Fisher, Doctoroff & Dobbs, 2002), the student's socioeconomic level (Verdine, et al. 2014), motivation (Middleton & Spanias, 1999), etc. Based on this hypothesis, the intention of this study was to verify the existence of a correlation between numerical discrimination and mathematical skills in first-year schoolchildren.

2. Methods

The project of this research was approved by the Ethics Committee of the São Paulo State University (Unesp). All participants had their consent terms signed by their parents and/or guardians. As pointed out in the project description and the target audience to be measured by the test, there was a need to exclude participants who had reports of neurological and cognitive problems.

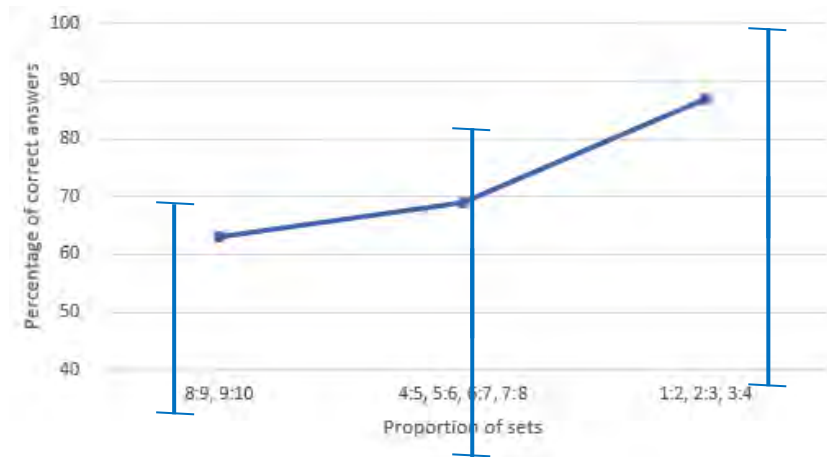
Two standardized tests were used. The first evaluated numerical discrimination by means the Panamath program (<https://panamath.org>). On the computer screen, at each trial, two sets of circles within juxtaposed rectangles are shown, each with a different number of elements. In each test trial, stimuli were exposed for 1200 ms and the participant was required to press one of two keys with yellow and blue discs stickers on the computer keyboard. The proportion of sets in each trial was established according to the parameters used in the study by Halberda and Feigenson (2008), namely: 1:2 - 2 trials, 2:3 - 2, 3:4 - 2, 4:5 - 2, 5:6 - 10, 6:7 - 10, 7:8 - 10, 8:9 - 14, 9:10 - 14, while the variation in the number of items was established between 1 and 14 points (Mazzocco et al., 2011). As a way of controlling non-numeric variables that could interfere with the children's responses, such as the size of the area occupied by the stimuli, the size of the "balls" was adjusted in 33% increments from the smallest size. The duration of this test was between 7 and 10 minutes.

The second test used was the Test of Early Mathematics Ability (TEMA 3), applied according to the protocol contained in the respective instructions. Each item presents the learning focus thematic, such as writing numerals, additions/subtracts, part-whole, mental number line, counting, enumeration, etc. In addition, each item has its own application instruction and which answer will be considered correct, which must be recorded on the answer sheet. The application took approximately 45 minutes, varying as the child progressed through each step of the procedure. The test starts with the entry point corresponding to the child's age (e.g., 3 - question 1; 4 - question 7; 5 - question 15; 6 - question 22; 7 - question 32; 8 - question 43). Ceiling represents an upper limit of performance and occurs when a child answers five consecutive items incorrectly. When a ceiling is reached, the examiner must stop testing. Likewise, the base represents a lower bound on the examinee's performance and consists of the five consecutive items that are answered correctly. Once the ceiling has been reached, a base must be established if the child has not made five consecutive mistakes. To establish a base in these cases, the examiner returns to the entry point and applies the previous items backwards until the child reaches their base or until item 1.

3. Results and discussion

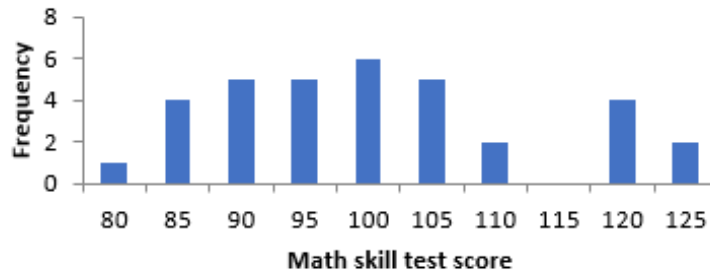
The first analysis to be carried out is the performance of children in the Approximate Numerical System activity. The results present that it was easier to discriminate smaller ratio sets, described by the 1:2, 2:3 and 3:4 ratios. The average presented from the children's answers on the test is given by 66.4% of correct answers, with a minimum of 45.5% and a maximum of 83.3%.

Figure 1. Relation between average percentage of correct answers in the numerical discrimination activity and its ratio sets with minimum and maximum margins.



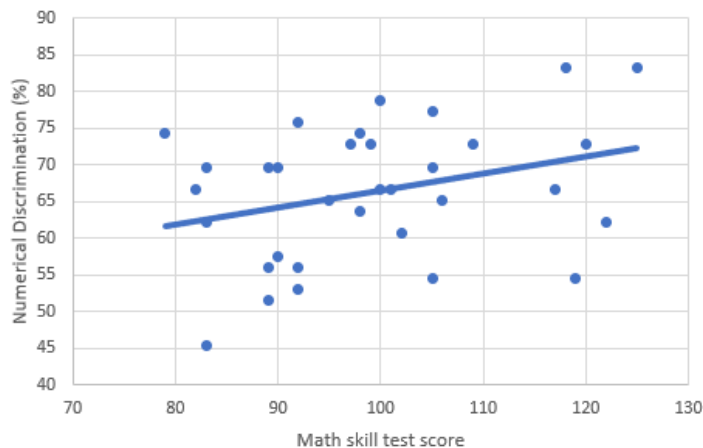
In this first moment, the possibility of a correlation between performance in the number sense activity and mathematical skills was considered, taking into account that the greater the child's sensitivity to differentiating the proportions between the sets, the better his performance in the solving mathematical problems addressed by TEMA-3. The test offers the score based on the total number of correct answers for each student. For the value of the math skills score to be correctly determined, it is necessary to relate the score to the age expressed in years and months of each child, as shown in the table in the TEMA-3 material.

Figure 2. Representation of results in the math skills test.



The histogram distribution shows that the highest frequency of responses is distributed between 95 to 105, showing a normal distribution of the collected data. With the results presented, it was a need to compare them both to analyze the hypothesis of the correlation among these two tests. The next figure shows the correlation between the percentage of correct answers in the numerical discrimination test and the math skills score. This correlation is in the margin of statistical significance, considering that the value of $p = 0.07$ and Pearson's correlation index at 0.31.

Figure 3. Representation of results in the math skills test.



The average of 63% (8:9, 9:10), 69% (4:5, 5:6, 6:7, 7:8) and 87% (1:2, 2:3, 3:4), presented in the activity performance of the approximate numerical system (Figure 1) shows a similarity with the data found in the literature (see Mazzocco et al., 2011). At first, it was considered the possibility of a correlation between the performance in the numerical discrimination test and the mathematical skills, taking into account that the greater the child's sensitivity to differentiate the proportions between the sets, the better their performance in the resolution of mathematical problems addressed by TEMA-3 would be.

The value found for this coefficient was 0.31, indicating a positive correlation between the variables, although not of great magnitude. However, other studies presented point to other factors that are related to the performance of the child's math skills, such as the mother's education, preschool education, the student's socioeconomic level and motivation (Koponen, 2007; Arnold, et al. 2002; Verdine, et al. 2014; Middleton & Spanias, 1999).

With the correlation value of 0.31 and $P = 0.07$ in the relationship between the percentage of correct answers in the numerical discrimination test and the total score of mathematical skills, it is believed that there is a possibility to predict the mathematical skills based on the performance in the task of numerical discrimination. This prediction is given above, considering that, as the value found in the percentage of correct answers increases, the higher the math skill score would be.

Although motivation, the student-teacher relationship, the social environment, among other factors influence the development of mathematical skills, numerical discrimination is essential for understanding the mechanisms underlying mathematical learning. Studies developed by Mazzocco et al. (2011) point to the relationship between numerical discrimination and mathematical results present during children's school development, as individual differences in cognitive abilities contribute to children's mathematical learning – and not only in mathematical performance at a given time, but also the growth trajectories in formal education. Therefore, conducting research such as the one presented here can contribute to the development of mechanisms for teachers to work with so that these difficulties are addressed.

Analyzing the predictive value – even if it is small – of the numerical discrimination test, it can be considered as an initial screening tool for identifying children at risk for math learning difficulties. In this way, the educator is able to identify this possible difficulty early and develop a reinforcement of these concepts before new knowledge is introduced.

It is necessary to consider these notes so that there is a continuity of these studies, so that we can relate this performance to other variables, such as social development and family background, response time and Weber fraction. Even though the conclusions of this article show possible relationships, important future investigations are still carried out so that this mathematical process can be considered clear and allow interventions aimed at improving the educational quality of children.

4. Conclusions

This study presents a moderate to low strength correlation between number sense and the development of math skills in first-grade children. The observed statistical significance did not reach the value established conventionally, but it is not negligible. As for the predictive power of one variable over the other, considering that the results show that numerical discrimination explains 7% of the assessed mathematical skills, this value does not seem sufficient to justify investments in the teaching of numerical discrimination, at least not as a single resource. It must be considered that several factors influence school performance in mathematics and that, in children with dyscalculia, the numerical approximation system has low accuracy (Bull & Johnston, 1997; Geary, 1993; Koontz & Berch, 1996). Studies on external influences, such as the child's social environment, are extremely important so that, from this point onwards, practices and the development of this knowledge in the beginning of the school years are reconsidered.

References

- Arnold, D. H., Fisher, P., Doctoroff, G., Dobbs, J. (2002) *Accelerating Math Development in Head Start Classrooms*. Journal of Educational Psychology, vol. 94, no 4, p. 762–70.
- Berch, D. *Making sense of number sense: implications for children with mathematical disabilities*. (2005) Journal of Learning Disabilities, vol. 38, no 4, p. 333–39.
- Bull, R., Johnston, R. S. (1997) *Children's Arithmetical Difficulties: Contributions from Processing Speed, Item Identification, and Short-Term Memory*. Journal of Experimental Child Psychology, vol. 65, no 1, p. 1–24.

- Corso, L. V., Dorneles, B. V. (2010) *Senso numérico e dificuldades de aprendizagem na matemática*. Rev. Psicopedagogia, v. 27, p. 298-309
- Geary, D. C. (1993) *Mathematical Disabilities: Cognitive, Neuropsychological, and Genetic Components*. Psychological Bulletin, vol. 114, no 2, p. 345–62.
- Halberda, J., Feigenson, L. (2008) *Developmental Change in the Acuity of the “Number Sense”: The Approximate Number System in 3-, 4-, 5-, and 6-Year-Olds and Adults*. Developmental Psychology, vol. 44, no 5, p. 1457–65
- Koontz, K. L. (1996) *Identifying Simple Numerical Stimuli: Processing Inefficiencies Exhibited by Arithmetic Learning Disabled Children*. Mathematical Cognition, vol. 2, no 1, p. 1–24.
- Koponen, T. et al. (2007) *Cognitive Predictors of Single-Digit and Procedural Calculation Skills and Their Covariation with Reading Skill*. Journal of Experimental Child Psychology, vol. 97, no 3, p. 220–41.
- Leibovich, T. et al. (2017) *From ‘Sense of Number’ to ‘Sense of Magnitude’: The Role of Continuous Magnitudes in Numerical Cognition*. Behavioral and Brain Sciences, vol. 40, p. e164.
- Mandler, G., Shebo, B. J. (1982) *Subitizing: An Analysis of Its Component Processes*. Journal of Experimental Psychology: General, vol. 111, no 1, p. 1–22
- Mazzocco, M. M. M., Feigenson, L., Halberda, J. (2011) *Preschoolers’ Precision of the Approximate Number System Predicts Later School Mathematics Performance*. PLoS ONE, vol. 6, no 9, p. e23749
- Middleton, J. A., Spanias, P. (1999) *A Motivation for Achievement in Mathematics: Findings, Generalizations, and Criticisms of the Research*. Journal for Research in Mathematics Education, vol. 30, no 1, p. 65.
- Mix, K. S., Huttenlocher, J., Levine, S. C. (2002) *Quantitative development in infancy and early childhood*. Oxford University Press.
- Moyer, R. S., Landauer, T. K. (1967) *Time Required for Judgements of Numerical Inequality*. Nature, vol. 215, no 5109, p. 1519–20
- Nunes, T., Bryant P. (1997) *Crianças fazendo matemática*. Porto Alegre: Artes Médicas.
- Sarama, J., Clements, D. H. (2009) *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.
- Verdine, B. N. et al. (2014) *Deconstructing Building Blocks: Preschoolers’ Spatial Assembly Performance Relates to Early Mathematics Skills*. Child Development, vol. 85, no 3, p. 1062–76.
- Xu, F., Spelke, E. S. (2000) *Large Number Discrimination in 6-Month-Old Infants*. Cognition, vol. 74, no 1, p. B1–11.