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SIMULATION OF DEVELOPMENTAL TRANSITIONS BELOW AND ABOVE FORMAL REASONING IN A NEURAL NETWORK MODEL

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Abstract

The Model of Hierarchical Complexity (MHC) is one post-Piagetian theory that shows that humans develop through stages beyond post-formal reasoning. At each developmental transition, simpler knowledge self-organizes into more complex knowledge without overwriting itself, however, the nature of transitions is poorly understood. We used neural networks to simulate this stage incremental process and to explore whether conclusions about transitions could be taken from the developing structure of the model, specifically when solving problems below and above formal reasoning. We simulated stage-wise human performance on the balance-beam test. The MHC analyzed the order of hierarchical complexity (OHC) of balance-beam tasks, identifying 4 OHC subtasks (9, 10, 11, and 12), each being solved by individuals at the following stages: concrete (stage-9), abstract (stage-10), formal (stage11) and systematic (stage-12). Hence, two stages below formal, where individuals operate with concrete information until they transform it into abstract information, and two stages above formal, where individuals operate with abstract information. In our method, we segregated the input set into the four disjoint OHC subsets. Then, we trained the minimal neural network model structure to solve each OHC subset separately. The best performing model for each OHC subset was selected and the evolving structure across sequential models was evaluated. Developmental transitions are represented by the recruitment of new neurons and connections from one OHC network to the next. First, results showed that segregating inputs by disjoint OHC led to the best performance of networks in the formal-order subtasks (torque difference calculus) among the literature. Second, transitions from concrete to abstract rely mostly upon an increase of memory resources of the existing connections. From abstract to formal and from formal to systematic, there is an increase in the number of neurons and connections. More than one transition pattern was found, which points towards the dynamic of self-organization. We either observed that there is an increase of 120% in both the number of neurons and connections from abstract to formal and a decrease of 50% from formal to systematic or that there is an increase of 80% in the number of neurons and connections, which maintains stable during the systematic performance. Limitations of this work concern the operations that were being learned at each OHC subtask, which conflict with the mathematical nature of neural network models. Even so, the scaling of network elements is worth exploring by simulating further OHC subtasks.

Keywords: *Stage of development, developmental transitions, simulation, neural networks, complexity*.

1. Introduction

Human cognition is incremental in nature and dependent upon learning, experience, and biological maturation (Commons & Pekker, 2008; Inhelder & Piaget, 1958; Klahr & Siegler, 1978). Developmental psychology theories soon suggested that human cognition passes through a series of steps (or stages). These stages are generally defined according to the problem-solving capabilities of individuals, either within or across domains (T. Dawson et al., 2003). "Stage" is assessed through sets of specifically designed problem-solving instruments that span through different domains (Giri et al., 2014). Since Piaget's experiments, these instruments have been improved, have progressively uncovered the building blocks of human cognition and have increased their power of explanation of how these building blocks develop throughout life (Dawson-Tunik et al., 2005). Since the 80's, this knowledge has given rise to computational models of cognitive development using neural networks, enriching the advances of the discipline of artificial intelligence (McClelland, 1988, 1995; McClelland & Jenkins, 1991). In fact, if we overlap the disciplines of developmental psychology and artificial intelligence, an immediate conclusion is that in order to maximize the similitude between artificial models and human cognition, one should adopt the longitudinal perspective-taking of learning and model the development of cognitive abilities (Leite, 2019; Leite & Rodrigues, 2018). The difficulty of this is that while stages of development have been identified by different theories with little variation, the dynamics of stage transition are still poorly understood. No theoretical or experimental references exist with sufficient detail that can inform simulation works (Maas & Hopkins, 2011). In fact, recent perspectives point towards the increasing validity of complex systems approach, where it is not so much a sequence of operations that is identified, but a whole network of interdependencies that gets modified (Mitchell, 1998). This present work addresses the developmental aspect of problem solving in an artificial learning model, exploring the dynamics of stage transition from the perspective of complex systems. We used neural networks to simulate this stage incremental process and to explore whether conclusions about transitions could be taken from the developing structure of the model, specifically when solving problems below and above formal reasoning.

The specific objective concerned the quantification of stage transition by observing the progression of the structure of the model as it learned increasingly complex problems. This was done by identifying the elements of the neural networks model structure while solving each order of complexity problem, with order of complexity being defined by the Model of Hierarchical Complexity (MHC). Three specific developmental transitions were targeted, concrete to abstract, abstract to formal, and formal to post-formal.

1.1. The Model of Hierarchical Complexity

One important developmental psychology theory is called Model of Hierarchical Complexity (MHC). It is a general-stage theory that has been extensively tested for assessing human development in different domains of knowledge and for creating different applications (https://www.dareassociation.org/). The MHC proposes that the assessment of developmental stage starts by measuring the complexity of problems to solve. This measure is a one-dimensional variable called the Order of Hierarchical Complexity (OHC) (Commons & Pekker, 2008). Problem complexity can be attributed a discrete value between 0 and 16. Hence, in total, 17 orders of hierarchical complexity have been found, which stand for identifying 17 stages of development. The highest problem complexity that an agent is able to successfully solve will be used for determining the stage of development of that agent, a human, non-human animal, or machine. The OHC of a problem is defined according to three simple axioms and further detailed in the literature (Commons, Gane-McCalla, et al., 2014; Commons, Li, et al., 2014). The MHC is reflected in the concepts and dynamics of complex systems theory. First, it differentiates horizontal and hierarchical complexity, assuming that only hierarchical complexity participates in stage transition (Commons & Pekker, 2008). This follows from the observation that only when two or more lower-order elements are combined, a higher-order stage emerges in a self-organizing way, reason why it has been difficult to trace the stage transition developmental route. There is evidence that stage is imprinted in the brain as a specific pattern of brain activation (Harrigan & Commons, 2014; Ribeirinho Leite et al., 2016), but this mapping has not been identified yet, let alone the process of transition from one mapping to the emergent one. Furthermore, the MHC has systematically observed in human behaviour that stage of development, as previously defined, acts as an attractor of the system. This means that the problems encountered in the environment will be perceived with a given complexity and solved accordingly. For example, a child and an adult facing the same situation will have different ways of perceiving and responding to it. The adult's stage, supposedly higher than the child's, acts as an attractor in the way that the adult is automatically led to be in the situation through the lens of their higher stage, unless they focus on assuming the child's lens to improve communication.

2. Methodology

We simulated the balance beam test, a developmental test applied to children. As the name indicates, the test is a balance beam that is presented to the child with different possible configurations of weights placed at different distances on each side of the fulcrum. The child should guess whether the beam would fall right, left, or balance if supporting blocks were removed from below. Some configurations are more difficult to guess than others because they display a more complex distribution of weights throughout the pegs. Extensive research on developmental psychology has shown that children respond to more difficult configurations of the test as they develop (Commons et al., 1995; Dawson-Tunik et al., 2010; Klahr & Siegler, 1978; Siegler & Chen, 2002). The MHC clearly characterizes difficulty in the balance beam problem by the mathematical operations that should be used to correctly predict the result of a given configuration (Dawson-Tunik et al., 2010), such as counting (at the concrete-stage-9), sum (at the abstract-stage-10), multiplication (at the formal-stage-11), and the distributive law (at the systematic-stage-12).

The simulation method we propose requires three steps. First, we simulated the balance beam with all possible configurations. Second, we grouped input vectors that represented problem configurations of the same order of complexity. Third, we created disjoint subsets of sequential OHC problems, following the order in which they are solved in the developmental trajectory. Fourth, we trained independent neural network models for solving each OHC problem, ensuring that the minimal model was found. We allowed the models to have a maximum of two layers, 20 units per layer, and 5 different patterns of connections where the simplest one was a pattern of feedforward connections. Fifth, we compared neural network models that solved adjacent OHC problems. Lastly, we quantified stage transitions based on the structural changes from one model to the next, both in terms of neurons and in terms of active connections.

3. Results

Among the best performing networks for each OHC sub-problem, some have been selected based on how their components (layers, units and connections) could be hierarchically organized across OHC. Networks performed with 100% accuracy for all stages but the systematic stage, where a slight decrease was observed. Table 1 represents the obtained results. Table 2 presents this quantification, done by a process of discretization.

Table 1. Numeric description of the selected networks.

OHC	**Option 1**		**Option 2**		**Option 3**	
	Nodes	Connections	Nodes	Connections	Nodes	Connections
Concrete stage 9	4	6	4	6	4	6
Abstract stage 10	4	8	4	8	4	8
Formal stage 11	12	135	11	103	8	88
Systematic stage 12	16 (2 layers)	195 (2 layers)	19 (2 layers)	194 (2 layers)	16 (2 layers)	195 (2 layers)

Table 2. Discretization of the transition process.

Transitions	**Option 1**		**Option 2**		**Option 3**	
	Nodes	Connections	Nodes	Connections	Nodes	Connections
First	0	2	0	2	0	2
Second	8	127	7	95	4	80
Third	4	60	8	91	8	107

In terms of how transitions are quantified in each option, given that in the first transition there was no change in the number of nodes, the second transition departs from a comparison with zero. Comparing to zero is an absolute increase. Hence, in each option, we are quantifying the increase in the number of in relation to the increase in the number of connections. In option 1, from abstract to formal there was an increase of 120% in both the number of neurons and connections and, from formal to systematic, there was a decrease of 50%. In option 2, from abstract to formal, there was an increase of approximately 80% to 90% in the number of neurons and connections, which remains stable during the systematic performance. In option 3, from abstract to formal, there was an increase of 80% in the number of neurons and connections, which, in the transition to the systematic stage, increased again by 50% in the number of nodes and in 25% in the number of connections.

4. Discussion and conclusion

A methodological proposal has been delineated to identify how a networked system represents developmental transitions at different orders of hierarchical complexity (OHC), or developmental stages as defined by the Model of Hierarchical Complexity (MHC). In this work, the number of units and the connectivity pattern have been experimented as variables for the network structure. A methodology was conducted to ensure that the minimal structural model was selected for solving each problem at each stage. After the training and selection processes, models for adjacent complexity problems were structurally compared and three structural progressions could be identified. Importantly, the model structure for each complexity problem was searched separately and independently, which allows to infer that the OHC is a valid and reliable measure to capture the complexity of problem solving and that neural networks capture this problem dimension in the dynamics of problem solving.

First, results showed that segregating inputs by disjoint OHC led to the best performance of networks in the formal-order subtasks (torque difference calculus) among the literature (M. Dawson & Zimmerman, 2003; Hofman et al., 2015; Maas et al., 2007; McClelland, 1988, 1995; Rijn et al., 2003; Shultz et al., 1994, 1994; Shultz & Schmidt, 1991). Until now, the torque difference problem has not been solved with 100% accuracy, nor with stability. Second, more than one transition pattern was found, which points towards the dynamic of self-organization. Actually, three structural progressions were identified, with similar accuracy results and with interesting similarities across transitions.

Regarding the similarities across all three progressions, two types of transitions were found: memory-based and operationally-based transitions. Memory-based occurred in the transitions below formal stages — from concrete stage 9 to abstract stage 10 — with no change in the number of nodes and a slight increase in the number of active connections, i.e., memory resources. This is aligned with the proposal of the Model of Hierarchical Complexity regarding the process of hierarchical integration as the dynamics of stage transition. Hierarchical integration refers to the idea that two or more lower order actions become object of non-arbitrary combination at the emergent order. This process has been compared to the process of working memory increase by chunking blocks of information, increasing working memory capacity throughout development (Duran et al., 2018; Kesteren et al., 2012). Below formal stages individuals operate with increasing concrete information, requiring an increase in working memory resources, until they transform it into abstract information. Differently, the present work suggests that operationally-based transitions occurred in the transition to and above the formal stages, relying upon a change in the structure, requiring more nodes and more connections. In these formal and post-formal stages, individuals become able to operate with abstract information. In this experiment, this was consistently shown by a boost in the recruited structural resources.

Analyzing each individual progression across transitions, interesting parallelisms are found, too, between the increase in the number of nodes and the increase in the number of connections. In the first scenario, there is an increase of 120% in both the number of nodes and connections from abstract to formal and a decrease of 50% from formal to systematic. In the second option, there is an increase of 80% in the number of neurons and connections, which maintains stable during the systematic performance. In the third option, there was an increase of 80% in the number of neurons and connections, which increased again by 50% in the number of nodes and in 25% in the number of connections in the transition to the systematic stage.

Limitations of this work concern the operations that were being learned at the concrete and abstract orders — counting and sum — which conflict with the mathematical nature of neural network models and justify that the model structure for these orders is the simplest. Even so, the memory-based transition is worth exploring in future work in the domain of pre-formal stages. Future work also concerns the simulation of more complex configurations, namely quadratic sum, to continue exploring the increase in structural complexity throughout post-formal simulations.

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