

THE ROLE OF DOMAIN-GENERAL FACTORS IN NUMERICAL PROCESSING IN EARLY CHILDHOOD

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Abstract

Numerical skills are a major cognitive foundation of mathematical learning, especially in young children. Even though multiple prior studies have examined the isolated role of domain-general factors in numerical processing, the complex relationships between multiple domain-general abilities and numerical abilities have not been tested together. For the first time, we tested spatial skills, visuospatial short-term memory, visuospatial working memory, attention, executive function, and fine motor skills to determine whether they represent distinct constructs and correlate with numerical abilities and the development of numerical abilities independently, or whether they cluster into integrated constructs. Using an extensive sample of children ($n = 339$) of varying ages (between 3 and 8 years old), we examined the influence of domain-general factors on numerical skills. All participants were introduced to the GiantLeap app, an app designed to conduct child development evaluations in non-controlled environments. The evaluation process is divided into two modules: a series of engaging tasks for the child and questionnaires for the parents. We used factor analysis to cluster tasks into latent variables to analyze the data. Based on the results, we used Structural Equation Modeling with numerical abilities as the outcome variable. First and foremost, we discovered that spatial skills—including visuospatial working memory, visuospatial short-term memory, and mental rotation—directly and strongly affect numerical skills. Executive functions also affect numerical skills but to a lesser extent. Notably, executive functions made a unique contribution to symbolic numerical skills, dissociating symbolic and non-symbolic numerical skills. Interestingly, attention (tested by hyperactive symptoms and commission rates) had no direct or indirect effect on numerical processing. Age affected multiple tasks, but not in the same manner. It had the strongest effect on omission rate and numerical processing, with a weaker effect on spatial abilities (except for mental rotation, which was directly affected by age). Age did not affect attention. These results indicate that multiple factors can affect numerical performance. This conclusion bears significant implications for the early diagnosis of learning disorders and intervention methods for these disorders.

Keywords: *Visuospatial short-term memory, visuospatial working memory, executive function, numerical processing, development.*

1. Introduction

Mathematical education is a relatively new cultural invention. However, it has been suggested that innate and primitive numerical abilities may be a marker for later mathematical abilities (Halberda & Feigenson, 2008). Specifically, evidence from infants, preschool children, and adults, as well as from non-human primates, has consistently shown that processing and manipulation of quantity are preverbal abilities based on defined neurological circuits (Ashkenazi et al., 2012). The approximate number system (ANS) hypothesis suggests that humans and non-human primates have a preverbal ability to intuitively understand approximate quantities and the relations between them. Later, after learning symbolic number words, these words (e.g., two) are mapped to non-symbolic quantity representation based upon the ANS (Halberda & Feigenson, 2008). In line with the ANS hypothesis, multiple studies have found modest positive correlations between accuracy in the ANS task, represented by the Weber fraction (the ratio between two compared approximate quantities that represents the highest individual discrimination ability), and mathematics, meaning that children and adults with better ANS acuity were found to show better math performances (see De Smedt et al., 2013, for narrative reviews).

However, other theories suggest that the symbolic numerical system develops independently from the approximate non-symbolic system and that in the early stages of development, only symbolic numerical representation is related to math achievements (Noel & Rousselle, 2011). According to these theories, math abilities should correlate more strongly with symbolic numerical abilities (tested mostly by symbolic

comparison) than with ANS abilities (tested by non-symbolic comparison). A recent meta-analysis confirmed this assumption; specifically, the study tested the relationships between children's and adults' mathematical abilities and symbolic and non-symbolic comparison abilities. The results revealed that the correlation with math abilities was significantly higher for the symbolic ($r = .302$) than for the non-symbolic ($r = .241$) magnitude comparison task, and also that the correlation between math abilities and numerical abilities (symbolic and non-symbolic) decreased very slightly with age (Schneider et al., 2017). However, the same meta-analysis concluded that magnitude comparison (symbolic and non-symbolic) is associated with mathematical competencies over the lifespan, signifying the role of symbolic and non-symbolic abilities in mathematical competencies (Schneider et al., 2017).

While numerical processing is a domain-specific cognitive foundation of children's mathematical learning, domain-general abilities also play a central role in mathematical learning and numerical processing. Multiple domain-general abilities, such as executive functions, attention, working memory, spatial abilities, and fine motor skills, each potentially play a role in the learning of mathematics and the performance of numerical tasks. Note that past studies focused primarily on the isolated effect of a single domain-general ability or on the impact of a few domain-general abilities on the ability to learn mathematics. A comprehensive understanding of the complex interrelations between multiple domain-general abilities and numerical task performance is still lacking (e.g., Gilmore & Cragg, 2018).

1.1. Project goals

Understanding the cognitive foundation of children's early numerical learning and the relationships between this foundation and domain-general factors has central educational significance.

Specifically, investigating children's early numerical learning and identifying markers for mathematical difficulties in very young children can lead to early interventions, even prior to the beginning of elementary school.

Study 1 - The development of symbolic and non-symbolic processing.

Study 2 - The role of visuospatial working memory in symbolic and non-symbolic processing.

Study 3 - Investigating the role of domain-general factors (spatial abilities, executive function, working memory, attention, and fine motor skills) in numerical processing in early childhood.

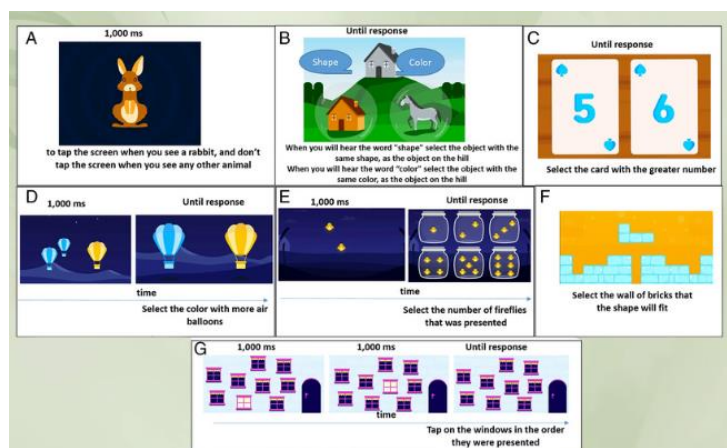
2. Methodology

2.1. Participants

Most of the participants were from the United States (80%), and others were native English speakers from other countries (such as Australia, Canada, and United Kingdom) that performed the GiantLeap diagnostic assessment. They did not receive any reward for participation other than feedback about their relative scores. Our dataset included 4,127 children who performed the diagnostic assessment between 2020 and 2021. Each child that was tested between the ages of 3 and 8 years and completed all the tasks, was included in our analysis: $N = 339$, 65% boys (Mean age girls 5.78, S.D. = 1.49; mean age boys 5.78, S.D. = 1.42).

2.2. Task

Figure 1. Demonstration of the task taken from the app. A Continuous performance task. B Color-Shape task. C Symbolic comparison D None- symbolic comparison. E Enumeration. F mental rotation. G Corsi Blocks (the presentation for reverse corsi blocks was the same but with different instructions).



3. Results

Figure 2. Correlations between accuracy in non-symbolic tasks with RT and accuracy of the symbolic task for younger children on the left (ages 3, 4, and 5), and older children on the right (ages 6 and 7). As can be seen, the correlation with RT was similar between ages. However, in terms of accuracy the correlation before elementary school was larger than the correlation after elementary school ($Z = 2.54, p = .005$).

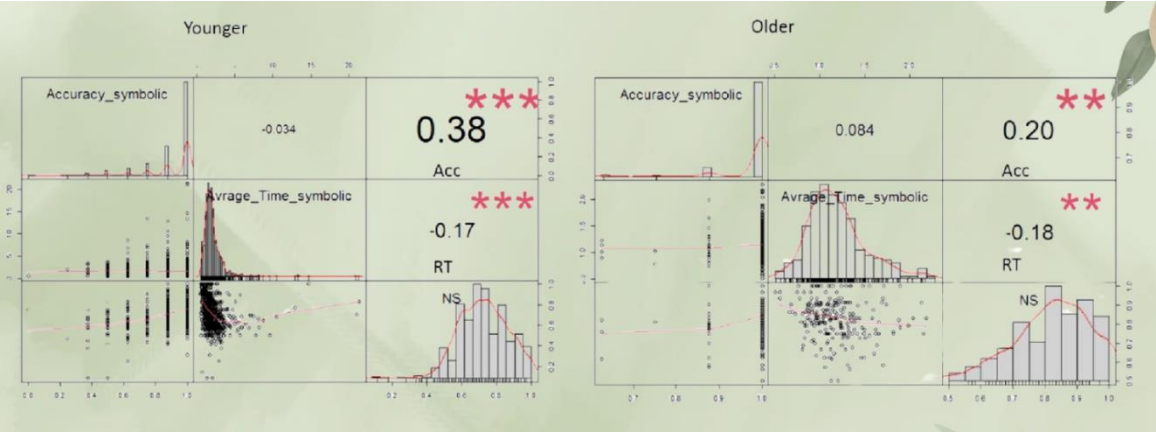


Figure 3. Correlations between RTs in symbolic task with RTs for subitizing and RTs for counting from the enumeration task for younger children on the left and for older children on the right. As can be seen, the correlation with subitizing was larger in the course of elementary school (ages 6 and 7, $r(208) = .37, p < .001$) than before elementary school (ages 3, 4 and 5, $r(696) = .17, p < .001$).

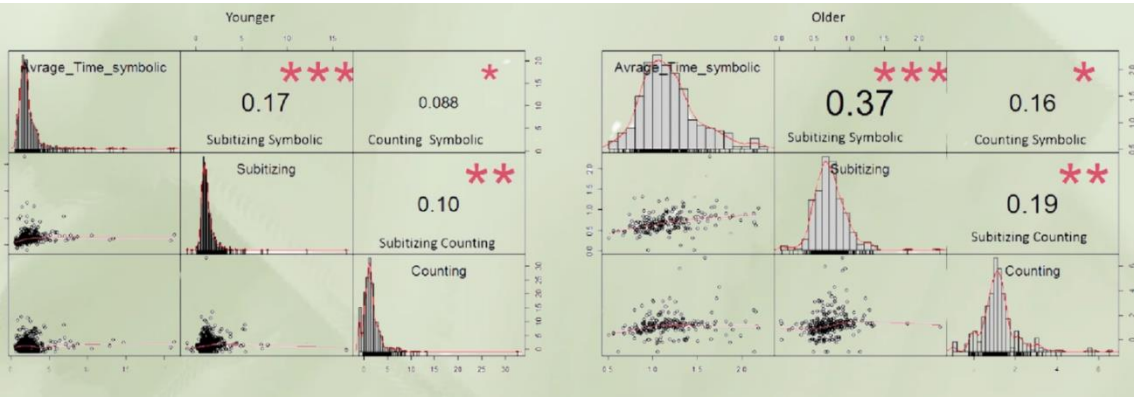


Figure 4. Developmental paths of the associations between visuospatial working memory and numerical processing, network analysis and SEM analysis.

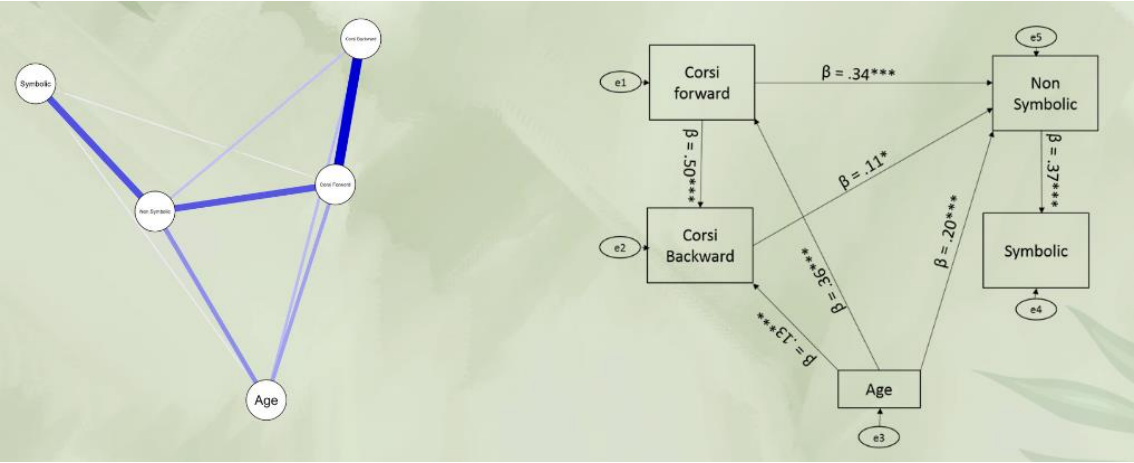
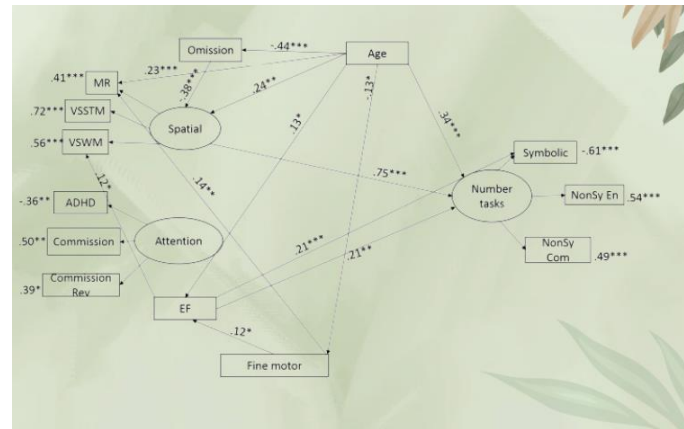


Figure 5. Final model based on the factor analysis results combined with modifications according to AMOS program suggestions. MR = mental rotation; VSSTM = visuospatial short-term memory; VSWM = visuospatial working memory; commission rev = commission reverse; EF = executive function. NonSy = non-symbolic. En = enumeration. Com = comparison * = $p < .05$, ** = $p < .01$, *** = $p < .001$



4. Discussion

One of the main questions that guided the present study was to understand how symbolic representation develops and whether symbolic representations are built upon approximate non-symbolic representation. The findings demonstrate that symbolic comparisons were associated with approximate non-symbolic representation most strongly prior to elementary school, while the correlations were weaker during elementary school.

On the other hand, correlations between subitizing and symbolic processing were found during elementary school but not prior to elementary school. This result suggests that symbolic representation of Arabic numerals changes during development. Before elementary school, there is a large resemblance between symbolic and non-symbolic approximate representation. However, during elementary school, two systems exist: one for symbolic representation, associated with subitizing, and another system for approximate non-symbolic representations.

One of the main goals that guided the present study was to understand the role of spatial abilities in numerical processing and the modulation effect of age on these relations. As expected, we found that the quantity comparison tasks are directly associated primarily with spatial short-term memory and, to a lesser degree, with spatial working memory. Because these two associations decrease with age, we suggest that younger children are using a spatial strategy more than verbally mediated strategies during numerical comparison tasks.

The associations were stronger between non-symbolic comparison and spatial abilities compared to symbolic comparison and spatial abilities. In fact, when including symbolic and non-symbolic comparison in one model, there was no direct link between symbolic comparison and spatial abilities. Similarly, in the unified model, age affected non-symbolic comparison abilities but not symbolic comparison abilities, demonstrating that symbolic representation is built upon non-symbolic representation.

Lastly, we discovered that spatial skills (composed of visuospatial working memory, visuospatial short-term memory, and mental rotation) directly and strongly affect numerical skills. Executive functions also affect numerical skills but to a lesser extent. Notably, executive functions made a unique contribution to symbolic numerical skills, dissociating symbolic and non-symbolic numerical skills. Interestingly, attention (tested by hyperactive symptoms and commission rates) had no direct or indirect effect on numerical processing. Age affected multiple tasks but not in the same manner. It had the strongest effect on omission rate and numerical processing, with a weaker effect on spatial abilities (except for mental rotation, which was directly affected by age). Age did not affect attention.

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