

A HYBRID MODEL OF EXECUTIVE FUNCTIONS, MATHEMATICAL ABILITIES AND TIME PROCESSING SKILLS

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Abstract

Time processing (TP) skills are fundamental functions in human beings, supporting different activities in daily life. These time-related abilities can facilitate both time monitoring and the sense of time, including time management and time orientation (Buzi, Eustache, Droit-Volet, Desauvay, & Hinault, 2024). An impairment in TP skills can affect the functioning of various life contexts, ranging from structured settings (e.g., school and work) to less structured ones (e.g., relationships, family, sports). According to the literature, deficits in TP skills are mainly observed in children with neurodevelopmental disorders, including attention deficit hyperactivity disorder (ADHD) and developmental dyscalculia (Moll, Göbel, Gooch, Landerl, & Snowling, 2016). This study aims to analyse the relationships between four latent constructs: executive functions (EF), mathematical abilities, time monitoring and sense of time (SoT). A total of 768 children (5-13 years, mean age: 8.61 ± 1.5 years; 50.8 % female) completed ad-hoc computerized time monitoring tasks, self and proxy-report questionnaires about the SoT, EF tests (e.g., auditory attention, auditory inhibition and working memory, and visual attention) and mathematical tests (e.g., mental calculations, numerical reasoning, numerical search and numerical line). A test was also administered to assess non-verbal reasoning. All assessments were conducted during school hours in group and individual sessions lasting 45 minutes. The proposed hybrid model, which combines confirmatory factor analysis (CFA) with pathway analysis, shows good fit indices (RMSEA = 0.05; CFI = 0.920; TLI = 0.896; SRMR = 0.06; Hu & Bentler, 1999). Using age and non-verbal reasoning as control variables, EF significantly predict time monitoring, and mathematical abilities significantly predict SoT. Age significantly predicts only SoT. The results suggest a significant connection between EF and mathematical abilities and time processing abilities. The implications of the results of this model are discussed from a dimensional point of view, considering how the potential presence of time-processing impairment in clinical populations, such as children with ADHD and Developmental Dyscalculia (e.g., Skagerlund & Träff, 2014). In future, it would be interesting to test the model for multiple groups by comparing typically developing populations with atypically developing populations.

Keywords: *Time processing skills, ADHD, developmental dyscalculia, executive functions, mathematical abilities.*

1. Introduction

Time processing (TP) is a multifaceted construct that involves fundamental skills supporting daily life activities. The literature has primarily focused on basic TP abilities related to the monitoring of time: estimating time intervals, reproducing time durations, and discriminating between different time intervals (Buzi et al., 2024). Such temporal abilities are typically assessed through ad-hoc computerized tasks that present stimuli in auditory and/or visual modalities (Zheng, Wang, Chiu, & Shum, 2022).

Basic time monitoring skills provide the foundation for more complex abilities included in the construct of sense of time (SoT), such as time management and orientation (Buzi et al., 2024). The ability to navigate into a typical day or week, plan activities, and meet deadlines is central to the development of an efficient SoT. Therefore, TP skills are essential for functioning in a wide range of everyday contexts. Impairments in TP may negatively affect functioning in both structured contexts, such as school and work, and less structured settings, including peer relationships and sports.

Deficits in time processing have been consistently identified in children with neurodevelopmental disorders, such as attention-deficit/hyperactivity disorder (ADHD) and specific learning disorder (SLD), especially developmental dyscalculia (DD; Moll et al., 2016; Skagerlund, Karlsson, & Träff, 2016).

According to literature, the core feature of ADHD appears to be deficits in cognitive processes, particularly executive functions (EF). TP skills are stronger in individuals with robust working memory, attention, and EFs (Ogden, Samuels, Simmons, Wearden, & Montgomery, 2018). Furthermore, TP deficits have been suggested as a potential endophenotype of ADHD (Hwang-Gu & Gau, 2015) with lower accuracy in time-related tasks (Walz Hapfelmeier, El-Wahsch, Prior, & Mecklinger, 2017).

Children with DD show difficulties in acquiring number facts and basic numerical skills (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Within this framework, the Theory of Magnitude (ATOM; Walsh, 2003) states that time, number, and space perception rely on a shared magnitude-processing system. According to this framework, individuals with DD show difficulties not only in numerical processing, but also in TP. Chronometric counting helps estimate durations and is tied to TP, but it may be difficult for individuals with DD because of their impaired counting abilities (Hurks & van Loosbroek, 2014).

This study aims to analyse the relationships between EF, mathematical abilities, and time monitoring/SoT measured with a new battery under validation in Italy.

1.1. Objectives

This study explores the relationships between EFs, mathematical abilities, time monitoring, and SoT in a school-aged community sample from a dimensional perspective. A hybrid model combining confirmatory factor analysis (CFA) and pathway analysis (Goldberger & Duncan, 1973) is used to examine latent constructs while controlling for age and non-verbal reasoning. This framework allows testing of theoretically grounded hypotheses and may support early identification of risk profiles and the development of targeted educational interventions.

2. Methods

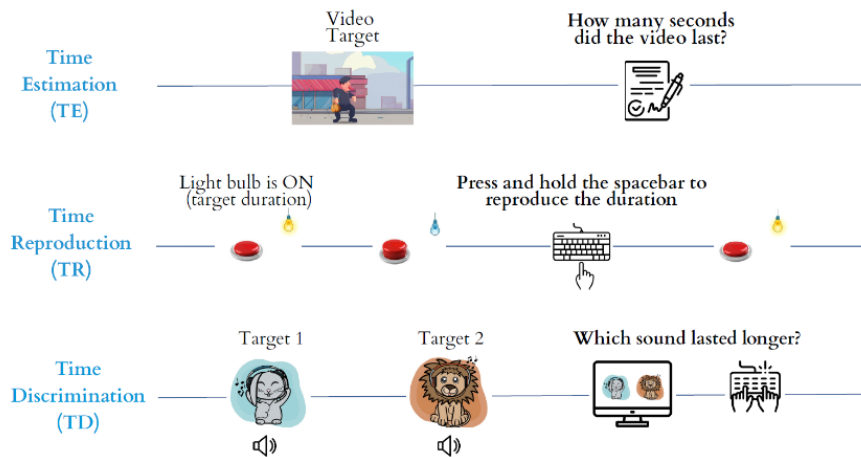
This cross-sectional study was conducted in primary and lower secondary schools in Italy. A total of 768 children (aged 5–13 years; $M = 8.61$, $SD = 1.5$; 50.8% female) were assessed between May 2024 and June 2025 during school hours, in group and individual sessions lasting approximately 45 minutes. Their parents and teachers also completed proxy-report questionnaires on the sense of time.

The tests for the assessment of TP skills are composed of ad-hoc computerized tasks and self- and proxy-report questionnaires regarding SoT. The computerized tasks are three: Time Estimation task, Time Reproduction task, and Time Discrimination task. The Time Estimation task (TE; based on Stojić, Topić, & Nadasdy, 2023) consists of two AI-generated videos, each lasting 30s. They are presented on a PC, accompanied by fast-paced music in one case and by more relaxed music in the other. Participants are asked to estimate the number of seconds corresponding to the duration of the videos presented, and the mean absolute deviation of the participant's response from the target duration is calculated. The Time Reproduction task (TR; based on Moll et al., 2016) was developed with the PsychoPy software (2023.2.3; Peirce et al., 2019). In each of the 12 trials, the target stimulus was a light bulb that turns on for a variable duration ranging from 2 to 12s. When the light bulb turns off, the participants were required to “turn it on” again for the same amount of time by holding down the spacebar on the PC. The score is calculated as the mean absolute deviation of the participant's response from the target duration. The Time Discrimination task (TD; Tobia, Rinaldi, & Marzocchi, 2018) consists of 36 trials, each including two visual and auditory stimuli of different durations. Each stimulus is presented as an animal hearing a sound lasting between 0.3 s and 7.5s, with increasing difficulty ratios. After both stimuli are presented, the participant must indicate which of the two animals heard the longer sound by pressing one of two keys on the computer keyboard. The score is calculated as the average of the last six reversals, referred to as the Ratio. Figure 1 summarizes the structure of the 3 ad-hoc computer tasks.

The self-report questionnaire on the SoT (STQ_Child; based on Porcelli et al., 2018) consists of 16 closed- and open-ended items that assess both time orientation and time management aspects. The proxy-report questionnaires for parents (STQ_Parent) and teachers (STQ_Teacher; Tobia et al., 2018) each consist of 10 items each, rated on a Likert scale ranging from 1 (never or rarely) to 4 (very often). The total score is obtained by summing the items responses.

EFs were assessed using standardized Italian subtests: selective auditory attention, working memory, and inhibitory control (Bisiacchi, Cendron, Gugliotta, Tressoldi, & Vio, 2005), as well as a visual attention (Marzocchi, Re & Cornoldi, 2021). Mathematical abilities were assessed using standardized Italian tests: mental calculations, numerical search, and numerical line (Biancardi, Nicoletti, & Bachmann, 2016), and numerical reasoning (Cornoldi, Mammarella, & Caviola, 2020). Non-verbal reasoning was also assessed (Kaufman & Kaufman, 2004).

Figure 1. Ad-hoc computerized tasks.



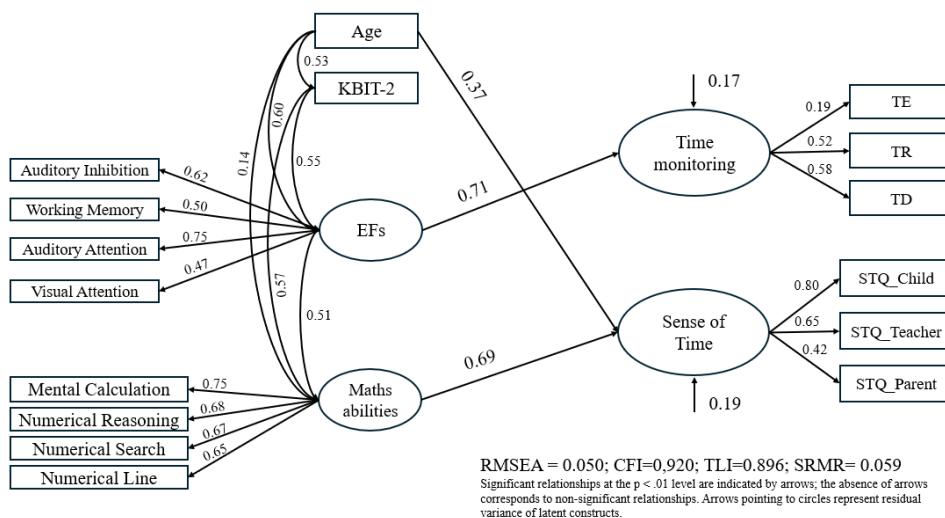
3. Analysis

Descriptive statistics were run using IBM SPSS Statistics (Version 29.02.0; IBM Corp., 2023). Based on the existing literature, a hybrid model was tested in MPlus (Version 6.11, Muthén & Muthén, 1998–2011) including four latent constructs: EFs, maths abilities, time monitoring and SoT. Age and non-verbal reasoning variables were included as predictors and possible covariates. We verified whether the variables grouped together consistently with respect to these four factors and observed the relationships between them. The model was estimated using the Maximum Likelihood (ML) method. To improve the model fit indices, the correlations recommended by the modification indices were fixed, according to theoretical background.

4. Results

The proposed hybrid model, which combines confirmatory factor analysis (CFA) with pathway analysis, shows good fit indices (Hu & Bentler, 1999). Using age and non-verbal reasoning as control variables, EFs significantly predict time monitoring, and maths abilities significantly predict SoT. Age significantly predicts only SoT. The results suggest a significant connection between EFs and maths abilities with TP skills (Figure 2).

Figure 2. Hybrid Model.



5. Discussion

The hybrid model presented showed good model fit indices, with fit indices within the ranges commonly accepted in the literature (Hu & Bentler, 1999). The latent variables of time monitoring and sense of time loaded significantly on their respective indicators, confirming the consistency of the theoretical grouping a posteriori. Similarly, the EFs and Math abilities tests showed adequate saturations on the corresponding factors. For both EFs and mathematical abilities, the indicators converge on the factor with non-identical loadings, highlighting the unity and diversity of the latent variables (Miyake & Friedman, 2012). EFs significantly predict the accuracy of time monitoring tasks. According to the literature, time monitoring is associated with EFs including working memory updating and inhibition (Ogden, Mackenzie-Phelan, Mongtomery, Fisk, & Wearden, 2019). Mathematical abilities significantly predict SoT. According to the literature, mathematical difficulties include deficits in counting processes (Butterworth, Varma, & Laurillard, 2011; Kaufman, 2008), which may be reflected in distortions in the perception, processing, and management of time (Cappelletti, Freeman, & Butterworth, 2011). The fixed correlations among age, non-verbal reasoning, executive functions, and math abilities are consistent with the literature (Arán Filippetti & Richaud, 2017).

Age shows a significant association with SoT. This result is consistent with the literature, which highlights how temporal sensitivity develops gradually with age, beginning to reach levels similar to those of adults as early as around 8 or 9 years of age (Droit-Volet, 2016). The absence of correlation between the two latent factors of time monitoring and SoT indicates that the two domains, although related in terms of temporal content, are conceptually distinct. SoT refers to a subjective and applicative dimension in which events are placed within an organised temporal framework, i.e. a more articulated representation that goes beyond the simple perception of durations (McCormack & Hoerl, 2017). Furthermore, several pieces of evidence indicate that explicit awareness of the passage of time can vary independently of the basic mechanisms of TP ability; the subjective experience of time is more influenced by cognitive, emotional and metacognitive factors that do not necessarily alter the ability to process the duration of time intervals (Droit-Volet, 2013). Overall, the model explains 83% of the variance in time monitoring and 81% of the variance in SoT, supporting the hypothesis of an association between EFs, math abilities and TP.

The dimensional approach used in this study enables us to consider what happens in clinical populations, including those with ADHD and developmental dyscalculia. In future, it would be interesting to test the model on multiple groups by comparing populations that develop typically with those with ADHD and developmental dyscalculia. Such studies could lay the groundwork for clinicians to assess and intervene in TP skills in children with neurodevelopmental disorders.

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